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# A Gram-Schmidt Space-Time Adaptive Canceller Using Adaptive Lattice Filters


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A new algorithm is presented for the implementation of an adaptive space-time canceller. This algorithm is a combination of the Gram-Schmidt and adaptive lattice filter orthogonalization techniques. The new algorithm is shown to have the potential of using far less hardware or equivalently, less numerical operations than other space-time canceller configurations.


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# A GRAM-SCHMIDT SPACE-TIME ADAPTIVE CANCELLER USING ADAPTIVE LATTICE FILTERS

## I. INTRODUCTION

An adaptive processor is a filter that bases its own design (its internal adjustment settings) upon estimated statistical characteristics of the input and output signals [1-3]. In particular, an adaptive spatial array processor weights the coherent output from each sensor (antenna) and adds them to form a receiving beam. For an adaptive array these weights may not be constant but rather can change as a function of the spatial properties of the noise field.

The objective of a space-time adaptive canceller (a special form of an adaptive processor) is to decorrelate or statistically orthogonalize the time samples of a group of auxiliary sensors from a given sensor (called the main channel). Figure 1 illustrates the general form of a space-time adaptive canceller. From this figure, the auxiliary channels are linearly weighted such that the output noise power residue of the main channel is minimized, which is equivalent to statistically orthogonalizing the auxiliary channels with respect to the main channel. Consider a more detailed illustration of the space-time adaptive canceller given in Fig. 2. The main channel input is designated by $x_0$, and the auxiliary channels are designated by $x_n$, $n = 1, 2, \ldots, N-1$. Let us set $N_{aux}$ equal to $N-1$. External signals are received at $N_{aux}$ distinct auxiliary sensors and sampled in time at equal intervals of $T$ seconds, using $L$ time delay taps per sensor. The main sensor is also sampled in time but may be delayed by some arbitrary time delay $\tau$.

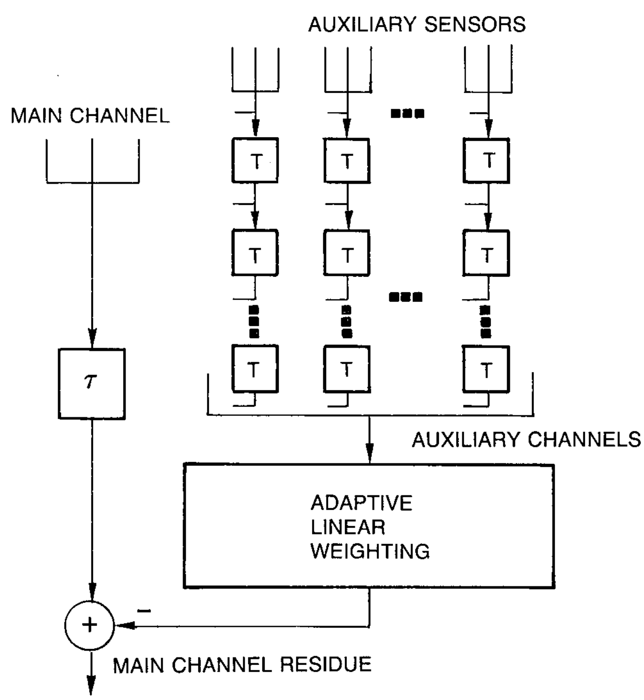


Fig. 1 — Generic space-time adaptive canceller

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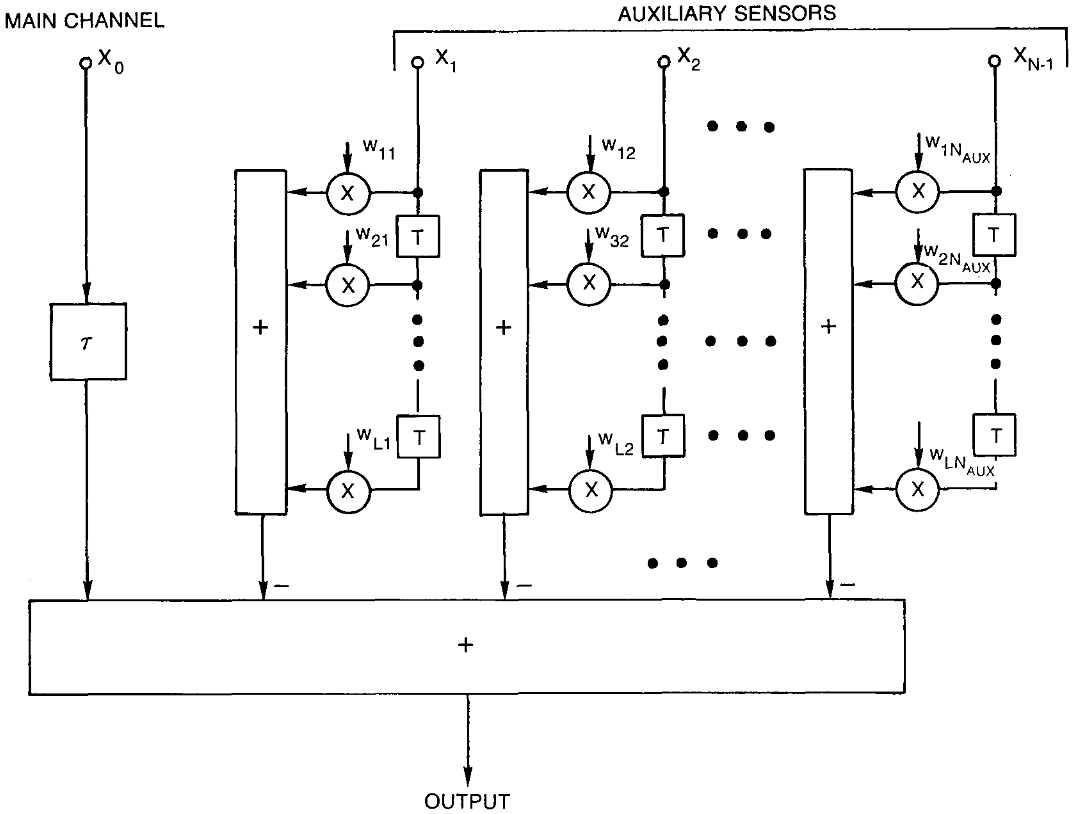


Fig. 2 — Space-time adaptive canceller

We define the $n$th auxiliary input data row vector $\mathbf{x}_n$ as

$$\mathbf{x}_n = (x_n(t), x_n(t - T), \cdots, x_n(t - (L - 1)T)) \tag{1}$$

and the optimal weighting row vector $\mathbf{w}_n$, of these $L$ taps as

$$\mathbf{w}_n = (w_{1n}, w_{2n}, \cdots, w_{Ln}). \tag{2}$$

We also define an $LN_{aux}$ length column vector, $\mathbf{X}$, of all auxiliary input data as

$$\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_{N_{aux}})^T, \tag{3}$$

where $T$ denotes the vector transpose operation, and an $LN_{aux}$ length optimal weighting column vector $\mathbf{W}$ as

$$\mathbf{W} = (\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_{N_{aux}})^T. \tag{4}$$

We define $R_{xx}$ to be the $LN_{aux} \times LN_{aux}$ input covariance matrix of the auxiliary inputs and $\mathbf{r}$ to be the $LN_{aux}$ length cross-covariance vector between the main and the $LN_{aux}$ auxiliary inputs. More formally,

$$R_{xx} = E\{\mathbf{X}^* \mathbf{X}^T\}, \tag{5}$$

$$\mathbf{r} = E\{\mathbf{X}^* x_0\}, \tag{6}$$

where $E\{\cdot\}$ denotes the expected value.

It can be shown [1-3] that $\mathbf{W}$ is the solution of the following linear vector equation

$$R_{xx}\mathbf{W} = \mathbf{r}. \tag{7}$$

One method of finding $\mathbf{W}$ is to use the Sampled Matrix Inversion (SMI) algorithm [4], which is illustrated in Fig. 3. For this technique, the sampled auxiliary covariance matrice $\hat{R}_{xx}$ and cross-correlation vector $\hat{\mathbf{r}}$ are formed, after which $\mathbf{W}$ is found by inverting $\hat{R}_{xx}$ and multiplying by $\hat{\mathbf{r}}$. This algorithm has been demonstrated to have a very rapid convergence rate. However, its disadvantages are its numerical complexity and possible instabilities related to taking the inverse at $\hat{R}_{xx}$.

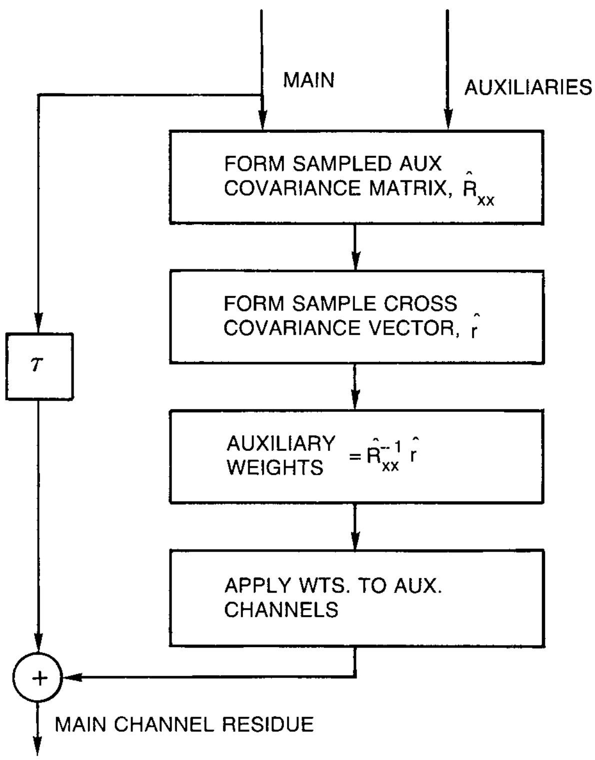


Fig. 3 — SMI canceller

In this report, we present a technique for implementing a space-time adaptive canceller by using Gram-Schmidt (GS) and adaptive lattice filter (LF) processing [5-10]. These techniques exhibit excellent performance simultaneously in arithmetic efficiency, stability, and convergence rate over other adaptive algorithms. The numerical stability of the algorithm is enhanced because it does not require the calculation of an inverse matrix as does the SMI algorithm.

In addition, we take advantage of the fact that the time-delayed samples in a given auxiliary channel are statistically stationary. That is, if we compute the $L \times L$ covariance matrix, $R_{x_n x_n}$, of the $n$th auxiliary channel where

$$R_{x_n x_n} = E\{\mathbf{x}_n^* \mathbf{x}_n^T\}, \tag{8}$$

then $R_{x_n x_n}$ is a hermitian toeplitz matrix that has the form

$$R_{x_n x_n} = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{L-1} \\ r_1^* & r_0 & r_1 & \cdots & r_{L-2} \\ r_2^* & r_1^* & r_0 & \cdots & r_{L-3} \\ \cdot & & & & \\ \cdot & & & & \\ r_{L-1}^* & r_{L-2}^* & r_{L-3}^* & \cdots & r_0 \end{bmatrix} \tag{9}$$

If the time samples are stationary in a given channel, then adaptive LFs [6-10] can be implemented resulting in a significant reduction in the number of numerical operations.

In Sections II and III, GS cancellation and adaptive LFs are reviewed respectively. In Section IV, the space-time GS canceller with adaptive LFs (GS/ALF) is described. In Section V, the space-time GS canceller is compared with the GS/ALF.

## II. GS CANCELLER

We briefly describe a GS canceller. Note that the open loop digital processing of the input data is fundamental to this implementation. Also the auxiliary channels to this processor consists of all of the time taps of the auxiliary antenna as seen in Fig. 1.

For a GS canceller the optimal weights formulated by Eq. (7) are not computed. The data in the input channels are filtered directly through an orthogonalization network as is demonstrated in the following discussion. However, the steady state output residue power in the main channel is the same as if the weights were calculated exactly by the use of Eq. (7) and applied to the input data set.

Consider the general $M$-input open loop GS canceller structure as seen in Fig. 4. Let $y_0, y_1, \cdots, y_{M-1}$ represent the complex data in the 0th, 1st, . . . , $M-1$th channels, respectively. We call the leftmost input $y_0$ the main channel, and we call the remaining $M-1$ inputs the auxiliary channels. The canceller operates so as decorrelate the auxiliary inputs one at a time from the other inputs by using the basic two-input GS canceller as shown in Fig. 5. For example, as seen in Fig. 4, in the first level of decomposition, $y_{M-1}$ is decorrelated with $y_0, y_1, \cdots, y_{M-2}$. Next, the output channel that results from decorrelating $y_{M-1}$ with $y_{M-2}$ is decorrelated with the other outputs of the first level GSs. The decomposition proceeds until a final output channel is generated. If the decorrelation weights in each of the two-input GSs are computed from an infinite number of input samples, then this output channel is totally decorrelated with the input: $y_1, y_2, \cdots, y_{M-1}$.

Let $y_n^{(m)}$ represent the outputs of the two input GSs on the $m-1$th level. Then outputs of the two-input GSs at the $m$th level are given by

$$y_n^{(m+1)} = y_n^{(m)} - w_n^{(m)} y_{M-m}^{(m)}, \quad \begin{matrix} n = 0, 1, \cdots, M-m-1, \\ m = 1, 2, \cdots, M-1. \end{matrix} \tag{10}$$

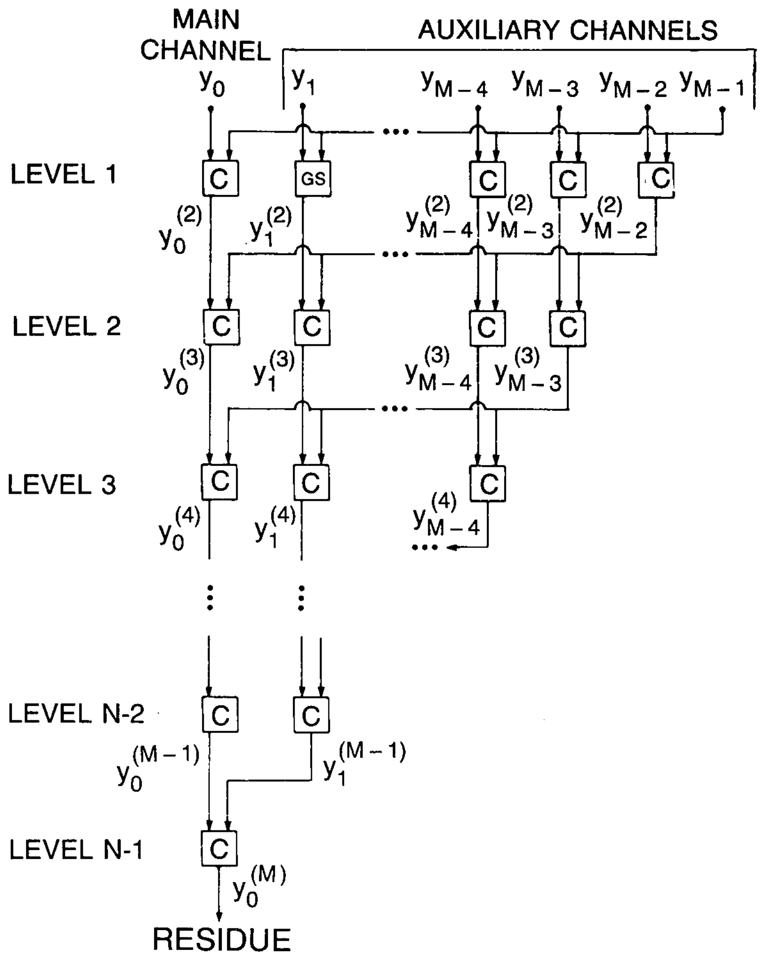


Fig. 4 — GS canceller

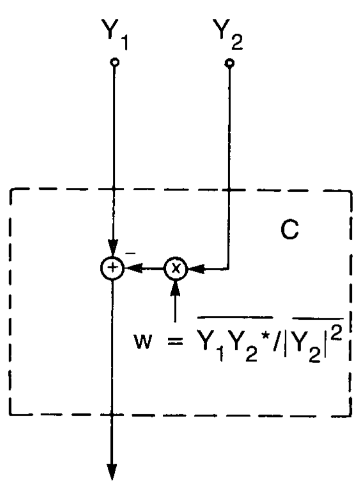


Fig. 5 — Basic two-input decorrelation processor or canceller

Note that $y_n^{(1)} = y_n$. The weight $w_n^{(m)}$, seen in Eq. (10), is computed so as to decorrelate $y_n^{(m+1)}$ with $y_{M-m}^{(m)}$. For $K$ input samples per channel, this weight is estimated as

$$w_n^{(m)} = \frac{\sum_{k=1}^{K} y_{M-m}^{(m)*}(k) y_n^{(m)}(k)}{\sum_{k=1}^{K} |y_{M-m}^{(m)}(k)|^2}, \tag{11}$$

where * denotes the complex conjugate and $|\cdot|$ is the magnitude. Here $k$ indexes the sampled data.

For the basic two-input canceller, the input to the right is the local main input unless otherwise noted.

## III. ADAPTIVE LATTICE FILTERS

A transversal filter is a method of processing time-stationary sequences. Its general form allows flexibility in meeting or attaining specified filter requirements and has been used extensively in digital filtering applications. Figure 6 shows the general form of a transversal filter. Here each input is time delayed in equal increments (normally at the sampling rate of the sample and hold circuit). Each of the $L$ input samples in time, $u(t)$, $u(t-T)$, . . . , $u(t - (L-1)T)$ are weighted. These weights are chosen in the optimal sense so that when all of the tapped time delayed samples are weighted and added to the main channel, then the main channel noise residue is minimized. Note that the main channel may be time delayed by $\tau$.

We set $\mathbf{u} = (u(t), u(t - T), \ldots, u(t - (L - 1)T))^T$. Then it can be shown that the optimal weights are the solution of the following vector equation:

$$R_{uu}\mathbf{w} = \mathbf{r}_{uv}, \tag{12}$$

where $R_{uu}$ is the $L \times L$ covariance matrix of the tapped time delays and $r_{uv}$ is the cross-covariance vector of length $L$ between the main channel and the tapped time delays.

If the samples of $u$ are statistically time stationary, then by definition

$$E\{u^*(t - kT)\, u(t - jT)\} = r_{k-j}, \tag{13}$$

i.e., the cross covariance between any two time delay taps is a function of only the relative time delay between the taps. The form of $R_{uu}$ is then a hermitian toeplitz matrix, where $r_{k-j}$, $k$, $j = 1, 2, \cdots, L$ are the elements of this matrix.

Instead of solving Eq. (12) directly for $\mathbf{w}$, an alternative solution is to orthogonalize the $u(t)$ input data and then solve for the optimal weights. Let the orthogonality transformation be the $L \times L$ matrix $A$. If we set $\mathbf{u}' = A\mathbf{u}$, then the optimal weighting vector $\mathbf{w}'$ must satisfy the following vector equation

$$R_{u'u'}\mathbf{w}' = \mathbf{r}_{u'v}, \tag{14}$$

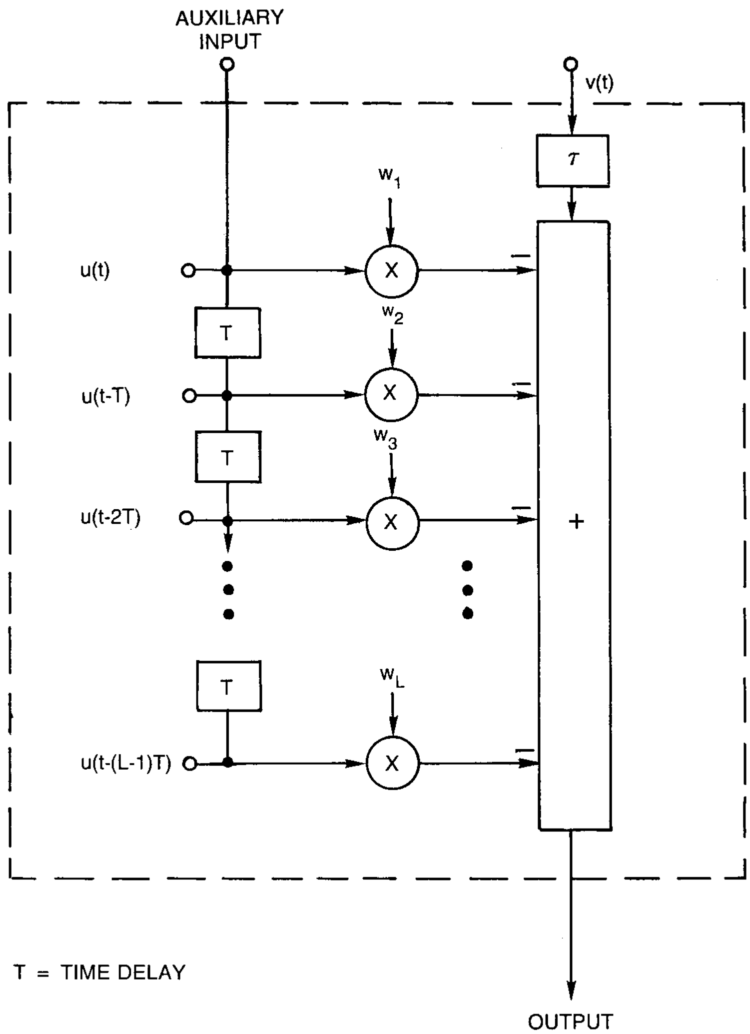


Fig. 6 — Transversal filter

where $R_{u'u'}$ is the $L \times L$ covariance matrix of the transformed tapped delays and $\mathbf{r}_{u'v}$ is the cross-covariance vector of length $L$ between the main channel and the orthogonalized inputs. Because the transformed tap delays are orthogonal, the matrix $R_{u'u'}$ is diagonal and hence the weights $\mathbf{w}'$ are easily derived.

Figure 7 shows an implementation of the method that statistically orthogonalizes and normalizes an $L$-tap input sequence. The basic two-input canceller building block shown in Fig. 4 has been used to orthogonalize an $N$-tap input sequence; i.e.,

$$E\{u'^{*}_{k}u'_{n}\} = 0; \quad k \neq n; \quad k, n = 1, 2, \cdots, L. \tag{15}$$

Because of its form, the filter shown in Fig. 7 is called an adaptive lattice filter (LF) [7].

We use the adaptive LF as a basic building block for the space-time canceller to be described in the next section.

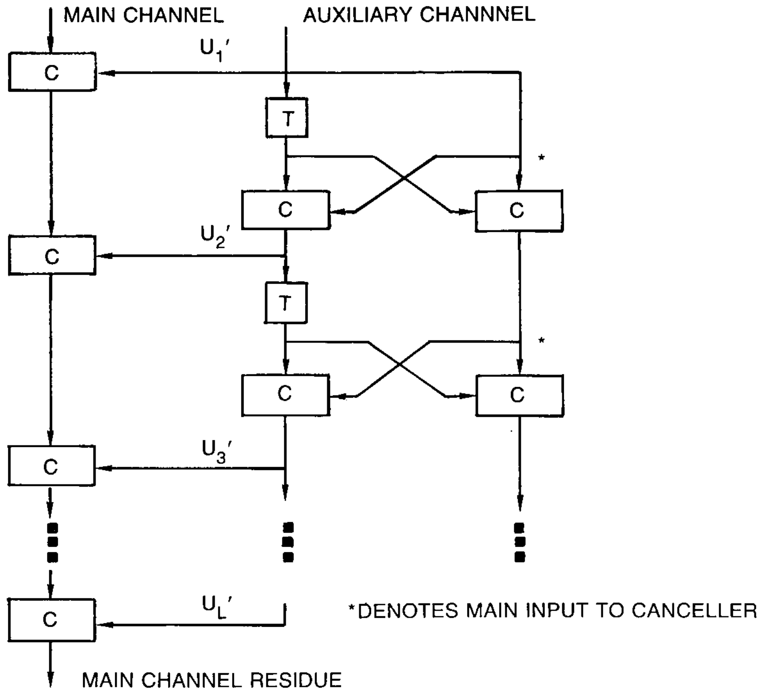


Fig. 7 — Adaptive lattice filter

## IV. GS CANCELLER WITH ADAPTIVE LFs

Figure 8 is the functional block diagram of the GS/ALF. The GS/ALF is an alternate implementation of the space-time filter illustrated in Fig. 2. However, it has several advantages that are discussed in the next section. Again, we assume that the data on a given channel are time stationary; i.e., the correlation functions are only a function of the time differences between channels. This allows us to use an adaptive LF in each channel. The number of time delay taps on each adaptive LF seen in Fig. 8 is $L$. The GS/ALF functions as follows.

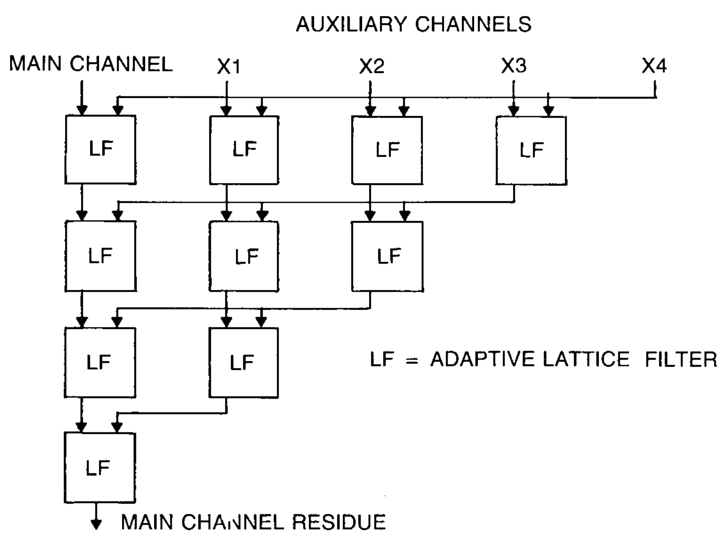


Fig. 8 — Gram-Schmidt with adaptive lattice filter
(GS/ALF) canceller configuration

First the $L$ tapped delays of the $x_{N-1}$ auxiliary channel are orthogonalized by using an adaptive LF, and the proper weightings are found for these $L$ orthogonal outputs such that these outputs are decorrelated (statistically orthogonalized) with respect to the $x_0, x_1, \ldots, x_{N-2}$ channels. In Fig. 8, the operation occurs at level 1 at each of the $N-1$ adaptive LFs.

Note that the resultant outputs from each of the $N-1$ LFs on level 1 are statistically time stationary. Hence, where initially we have $N-1$ time-stationary auxiliary antenna channels to decorrelate with the main channel, we have reduced the number to $N-2$ time-stationary auxiliary antenna channels.

Next, the $L$ tapped delays of the output from the far-right adaptive LF on level 1 is orthogonalized. The $L$ outputs are properly weighted to decorrelate them from the other $N-2$ outputs of the first level adaptive LF. Hence after level 2 of the decorrelation process, we have $N-3$ time-stationary antenna channels to decorrelate with the main channel.

It is apparent that the above operations can be repeated until there are no auxiliary channels to decorrelate and the main channel is completely decorrelated from the time taps of the auxiliary channels.

The orthogonalization of the time-stationary data of the rightmost channel that occurs at each level of the GS/ALF is common to all of the adaptive LFs at a given level. Hence, this need not be duplicated for each adaptive LF at a given level. A more numerically efficient implementation of the GS/ALF is illustrated in Fig. 9 where this redundancy is taken advantage of in the processing structure. Note for this example $L = 3$, $N = 4$.

## V. COMPARISON

Note that the GS/ALF canceller is *not* an implementation of the adaptive weighting indicated by Eq. (7). However, it is a canceller which uses less two-input building block cancellers and numerical operations than the "full up" GS. Simulation studies are now being performed to quantify the GS/ALFs cancellation and convergence performance and will be documented in a future report.

We compare the total number of two-input building block cancellers needed for GS/ALF implementation $\mathrm{NC}_{GS/ALF}$ with the total number of two-input cancellers needed for a "full up" GS implementation $\mathrm{NC}_{GS}$. Here we assume that there are $N$ antenna channels (main and auxiliaries) and $L$ taps per channel.

For the complete GS implementation of this space-time canceller we can show that

$$\mathrm{NC}_{GS} = \frac{1}{2}\,[(N-1)L+1]\,(N-1)L \approx \frac{1}{2}(NL)^2. \tag{17}$$

For the GS/ALF implementation, we can show

$$\mathrm{NC}_{GS/ALF} = (N-1)[L\ \ (.5N+2)-3] \approx \frac{1}{2}N^2L. \tag{18}$$

Hence

$$\frac{\mathrm{NC}_{GS/ALF}}{\mathrm{NC}_{GS}} = \frac{1}{L}, \tag{19}$$

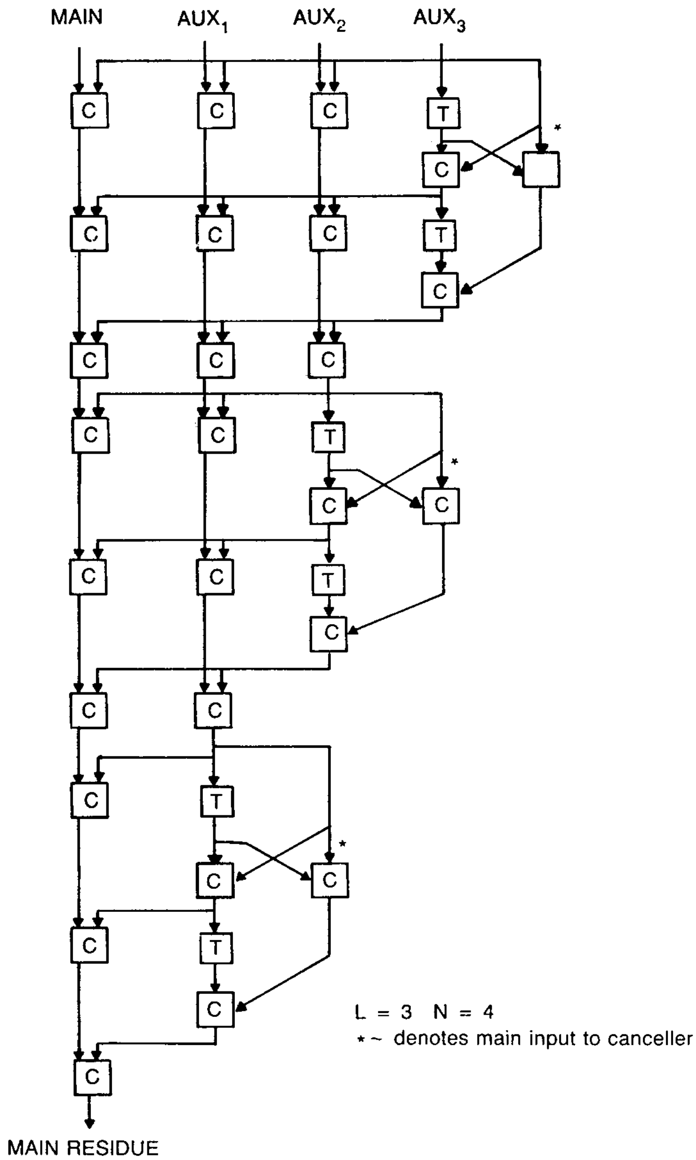


Fig. 9 — Efficient implementation of GS/ALF

or the GS/ALF canceller implementation uses approximately $1/L$ fewer building block cancellers than the "full up" GS canceller implementation. For example, if there are five time taps, the GS/ALF uses approximately one-fifth as many building block cancellers as the GS canceller.

It can also be shown that the numerical efficiency (total number of complex multiplications) of GS/ALF canceller to the GS canceller is also inversely proportional to the number of time taps.

## VI. REFERENCES

1. S.P. Applebaum, "Adaptive Arrays," *IEEE Trans. Antennas Propag.*, **24**(5), 585-598 (1976); also Syracuse Univ. Res. Corp., Rep. SPL TR 66-11, Aug. 1966.

2. B. Widrow, P.E. Mantey, L.J. Griffiths, and B.B. Goode, "Adaptive Antenna Systems," *Proc. IEEE* **55**, 2143-2159 (1967).

3. R.A. Monzingo and T.W. Miller, *Introduction to Adaptive Arrays*, (John Wiley and Sons, New York, 1980), Chap. 8.

4. I.S. Reed, J.D. Mallett, and L.E. Brennan, "Rapid Convergence Rate in Adaptive Arrays," *IEEE Trans.* **AES-10**, 853-863 (1974).

5. B.L. Lewis and F.F. Kretschmer, Jr., published work of limited distribution, dating back to Feb. 1974.

6. M.A. Alam, "Orthonormal Lattice Filter—A Multistage, Multichannel Estimation Technique," *Geophys.* **43**, 1368-1383 (1978).

7. B. Friedlander, "Lattice Filters for Adaptive Processing," *Proc. IEEE* **70**(8), 829-867 (1982).

8. J.M. Delosme and M. Morf, "Mixed and Minimal Representations for Toeplitz and Related Systems," in *Proc. 14th Asilomar Conf. Circuits, Systems & Computers* (Pacific Grove, Calif., Nov. 1980), pp. 19-24.

9. B. Friedlander, M. Morf, T. Kailath, and L. Ljung, "New Inversion Formulas for Matrices Classified in Terms of Their Distance from Toeplitz Matrices," *Linear Algebra and Its Applications* **27**, 31-60 (1979).

10. T. Kailath, A. Vieira, and M. Morf, "Inverses of Toeplitz Operators, Innovations, and Orthogonal Polynomials," *SIAM Rev.* **20**(1), 106-119 (1978).